

Name: ANSWER KEY

Date: _____

Per.: _____

FINAL REVIEW – Basic Geometry Vocabulary

Euclid:

- the father of Geometry
- studied by Abraham Lincoln
- built an *axiomatic* system of Geometry
 - based on **axioms** – statements accepted as true
 - ex: A straight line segment can be drawn joining any two points.

	Description	Figure	Symbol
★ point	describes a location; zero dimensions		P or Point P
★ line	a collection of points along a straight path with no endpoints; one dimension (length)		\overleftrightarrow{AB} or \overleftrightarrow{BA}
★ plane	a flat surfaces that extends indefinitely; two dimensions (length and width)		Plane EFG or Plane T
ray	a collection of points along a straight path with one endpoint which extends indefinitely in one direction; one dimension (length)		\overrightarrow{PQ}
line segment	a collection of points along a straight path with two endpoints; one dimension (length) *measurable		\overline{XY} or \overline{YX}
angle	two rays that meet at a point (this point is the vertex) *measurable		$\angle ABC$

★ These are the UNDEFINED TERMS in Geometry...
many vocabulary words we use depend on one or more of these 3 words.

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Geometry

Directions: I HIGHLY recommend answering these on a separate sheet of paper.

segment, ray \overrightarrow{PA} , line \overleftrightarrow{PQ} , ray \overrightarrow{QP}

1. Describe what each of these symbols means: \overline{PQ} , \overrightarrow{PQ} , \overleftrightarrow{PQ} , \overrightarrow{QP} .

2. Sketch a line that contains point R between points S and T . Which of the following are true?



A. \overleftrightarrow{SR} is the same as \overleftrightarrow{ST} . \checkmark

B. \overleftrightarrow{SR} is the same as \overleftrightarrow{RT} . \checkmark

C. \overleftrightarrow{RS} is the same as \overleftrightarrow{TS} . \checkmark

D. \overleftrightarrow{RS} and \overleftrightarrow{RT} are opposite rays. \checkmark

E. \overleftrightarrow{ST} is the same as \overleftrightarrow{TS} . \checkmark

F. \overleftrightarrow{ST} is the same as \overleftrightarrow{TS} . \checkmark

Decide whether the statement is true or false.

3. Points A , B , and C are collinear. \checkmark

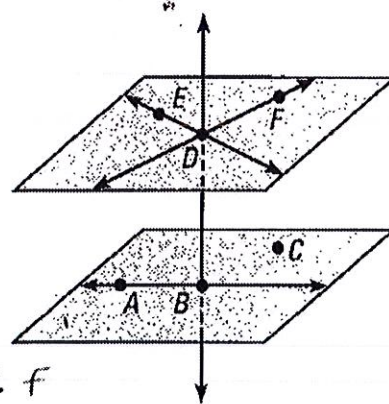
4. Points A , B , and C are coplanar. \checkmark

5. Point F lies on \overleftrightarrow{DE} . \checkmark

6. \overleftrightarrow{DE} lies on plane DEF . \checkmark

7. \overleftrightarrow{BD} and \overleftrightarrow{DE} intersect. \checkmark

8. \overleftrightarrow{BD} is the intersection of plane ABC and plane DEF . \checkmark



Short Answer:

1. What are the undefined terms?

Point, line, plane

2. Why will two points ALWAYS be collinear? Why will three points always be coplanar?

① B/c there is always a unique line that will connect any two points

② B/c for any 3 points, there exists a plane that contains all 3 points

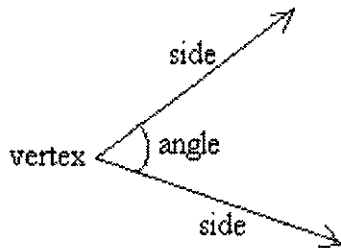
3. In what way(s) was Euclid influential?

- built/organized the system of Geometry based on a set of axioms (statements accepted as true w/o proof)

- Lincoln studied Euclid to build his skills in logic to use toward making a good, structured argument

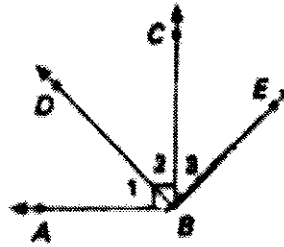
FINAL REVIEW – Angles

What is an angle?



*the angle is the space between the two sides... a portion of 360°

How do you name an angle?



$\angle 1$ can also be named $\angle ABD$ or $\angle DBA$

$\angle 2$ can also be named $\angle DBC$ or $\angle CBD$

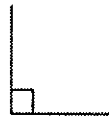
notice that the VERTEX is always the letter in the middle

How can you describe an angle?

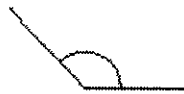
Acute angle
less than 90°



Right angle
 $= 90^\circ$



Obtuse angle
between 90°
and 180°



Straight line
 $= 180^\circ$



Reflex angle
greater than
 180°



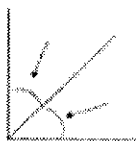
Angle relationships



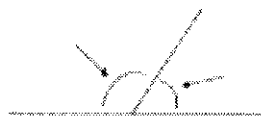
adjacent angles



vertical angles



complementary angles

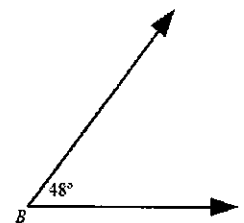
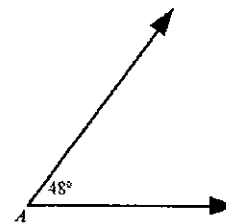


supplementary angles

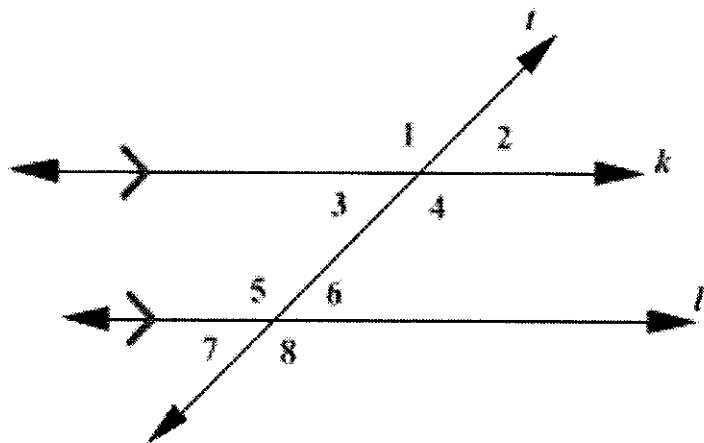
Congruent Angles

Congruent angles have the same angle measure.

$\angle A \cong \angle B$ because the measure of both angles is 48° .



Parallel Lines cut by a Transversal



In the diagram to the left, line l and line k are parallel. Line t is a **transversal** because it cuts through both lines.

Key Points:

1. Vertical angles are congruent.

Ex: $\angle 1 \cong \angle 4$ and $\angle 7 \cong \angle 6$

2. Alternate interior angles are congruent.

Ex: $\angle 3 \cong \angle 6$ and $\angle 5 \cong \angle 4$

3. Corresponding angles are congruent.

Ex: $\angle 1 \cong \angle 5$ and $\angle 8 \cong \angle 4$

4. Same side interior angles are supplementary.

Ex: $m\angle 6 + m\angle 4 = 180^\circ$

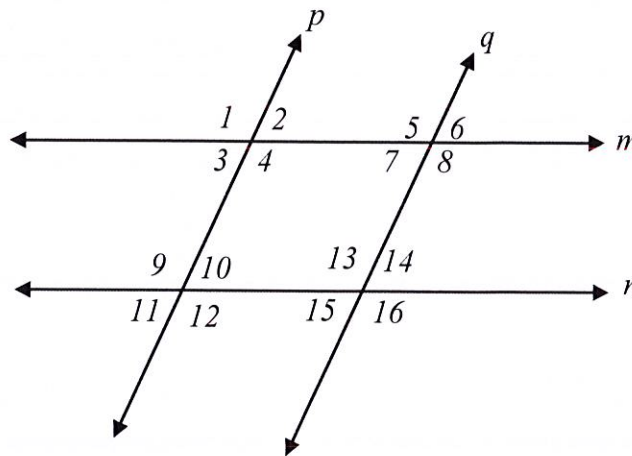
5. Same side exterior angles are supplementary.

Ex: $m\angle 8 + m\angle 2 = 180^\circ$

6. Linear pairs are supplementary.

Ex: $m\angle 1 + m\angle 2 = 180^\circ$

FINAL REVIEW – Angles: Problem Set



Using the diagram above and given that line m is parallel to line n :

1. Name a pair of alternate interior angles.

Many answers Ex: $\angle 8 + \angle 13$, $\angle 7 + \angle 14$, $\angle 4 + \angle 9$

2. Name a pair of corresponding angles.

Many answers Ex: $\angle 2 + \angle 10$, $\angle 11 + \angle 3$

3. Name a pair of vertical angles.

Many answers Ex: $\angle 1 + \angle 4$, $\angle 13 + \angle 16$, $\angle 15 + \angle 14$

4. If $m\angle 11 = 80^\circ$, what is the measure of $\angle 10$? How do you know?

$m\angle 10 = 80^\circ$ b/c they are vertical angles, which are congruent to each other

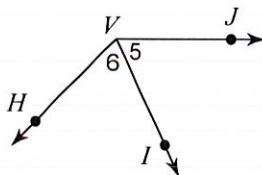
5. If $m\angle 11 = 80^\circ$, what is the measure of $m\angle 12$? How do you know?

$m\angle 12 = 100^\circ$ b/c linear pairs form supplementary angles

6. If $m\angle 2 = 65^\circ$ and $m\angle 5 = 115^\circ$, are lines p and q parallel? Explain why or why not.

Yes! These are same side interior angles. If they are supplementary, the lines are parallel. ($65^\circ + 115^\circ = 180^\circ$)

6. Using the diagram below, name three angles with the vertex V .



$\angle JVI$, $\angle HVI$, $\angle HVJ$,
 $\angle 5$, $\angle 6$

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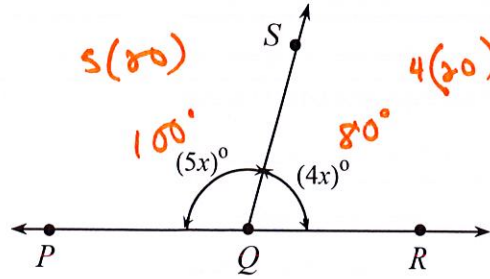
In the figure on the right, points P , Q and R are collinear. What is the measure of $\angle RQS$?

- A. 40° B. 20° **C. 80°**

- D. 50° E. 100°

$$9x = 180$$

$$x = 20$$



8.

If $\angle A$ and $\angle B$ are complementary, $\angle B$ and $\angle C$ are supplementary, and $m\angle A = 64^\circ$, then what is the measure of $\angle C$?

- A. 64° B. 180° C. 26° D. 90°

$$m\angle A + m\angle B = 90^\circ$$

$$64 + m\angle B = 90^\circ$$

$$m\angle B = 26^\circ$$

$$m\angle B + m\angle C = 180^\circ$$

$$26 + m\angle C = 180$$

- E. 154°**

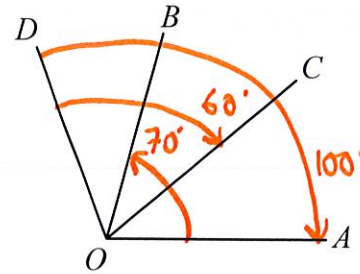
$$m\angle C = 154^\circ$$

9.

In this figure, $m\angle AOB = 70^\circ$, $m\angle COD = 60^\circ$, and $m\angle AOD = 100^\circ$. What is $m\angle COB$?

- A. 10° B. 65° C. 35°

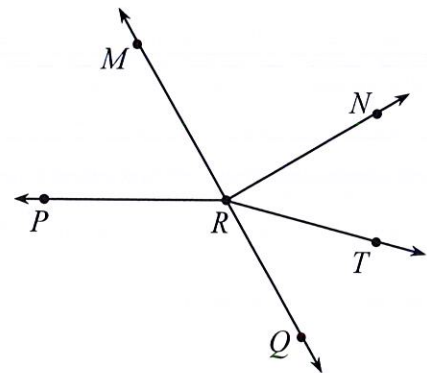
- D. 60° **E. 30°**



Problems 14-16: Refer to the figure on the right, in which M , R and Q are collinear and $m\angle MRN = 90^\circ$:

- C** 14. Which of the following is a *straight angle*?

- A. $\angle MRN$ B. $\angle PMR$ C. $\angle MRQ$
D. $\angle PRN$ E. $\angle NTR$



- B** 15. Which of the following is an *obtuse angle*?

- A. $\angle MRQ$ B. $\angle PRN$ C. $\angle NTR$ D. $\angle MRN$ E. $\angle PMR$

- A** 16. Which of the following angles is *adjacent* to $\angle NRT$?

- A. $\angle QRT$ B. $\angle MRT$ C. $\angle PRM$ D. $\angle PRN$ E. $\angle PMR$

FINAL REVIEW – Midpoint, Distance, and Partitioning a Line Segment

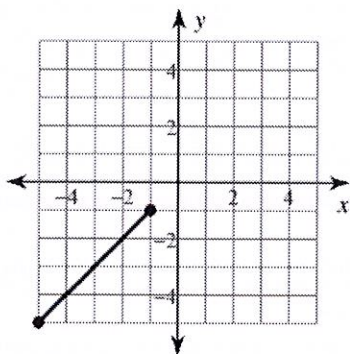
The Formula:

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Explained:

The midpoint is the average of the x-coordinates and the average of the y-coordinates.

Find the midpoint of each line segment.



Coordinates of the endpoints:

$(-5, 5)$

$(-1, -1)$

$$\left(\frac{-5 + -1}{2}, \frac{-5 + -1}{2} \right)$$

$$(-3, -3)$$

11) $(2, 4), (1, -3)$

$$\left(\frac{2 + 1}{2}, \frac{4 + -3}{2} \right)$$

$$(1.5, 0.5)$$

12) $(-4, 4), (-2, 2)$

$$\left(\frac{-4 + -2}{2}, \frac{4 + 2}{2} \right)$$

$$(-3, 3)$$

The Formula:

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Explained:

Derived from the Pythagorean Theorem

Find the **length** of the line segment with the following endpoints OR find the **distance** between the following two points: $(3, 7)$ and $(2, 9)$.

$$\sqrt{(2 - 3)^2 + (9 - 7)^2}$$

$$\sqrt{(-1)^2 + (2)^2}$$

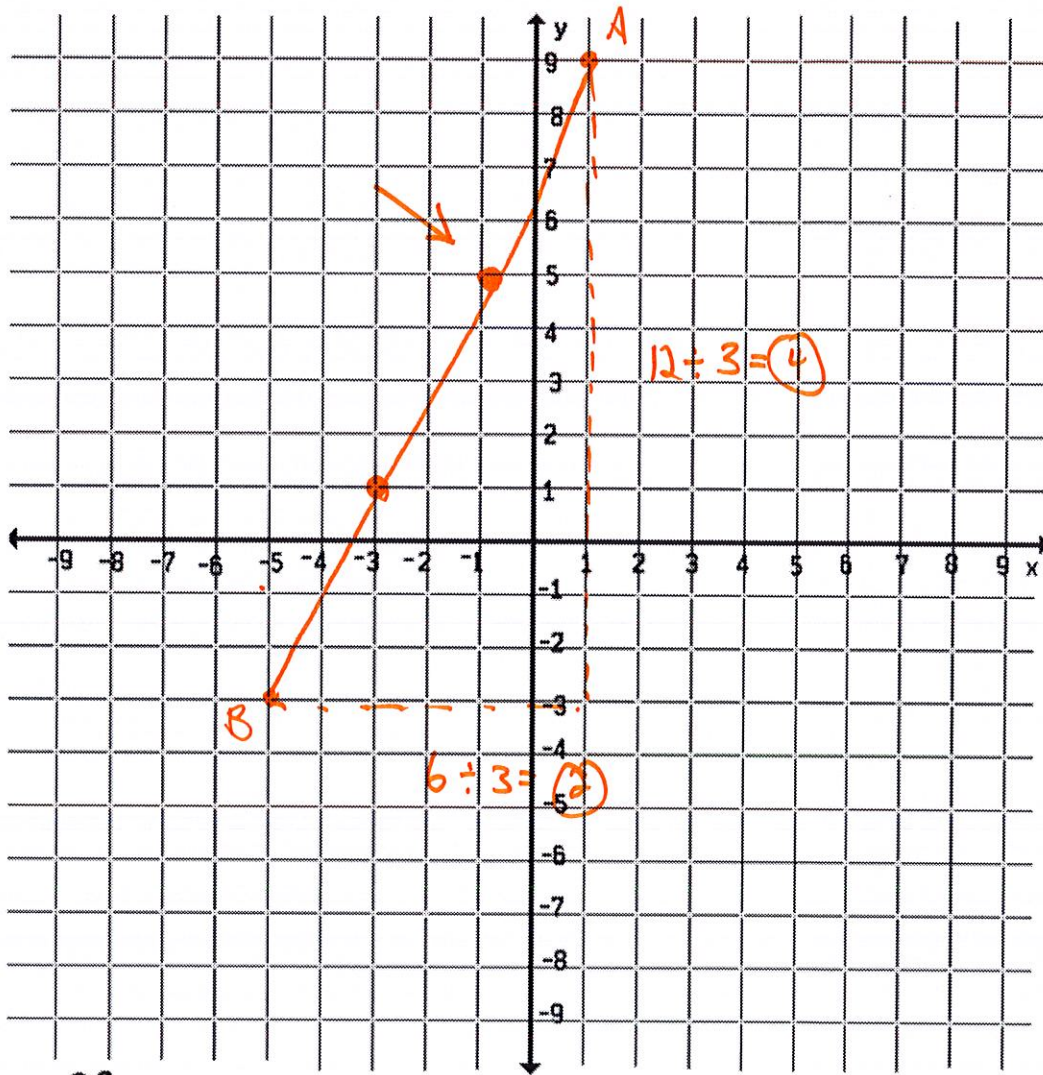
$$\sqrt{1 + 4}$$

$$\sqrt{5}$$

Partitioning a Line Segment

3 parts

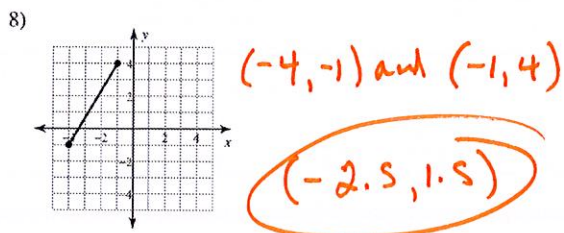
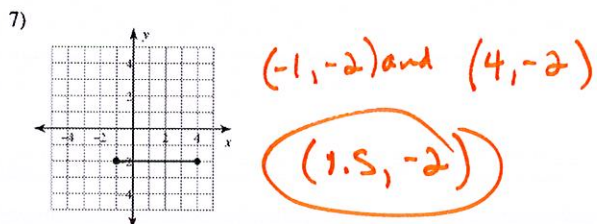
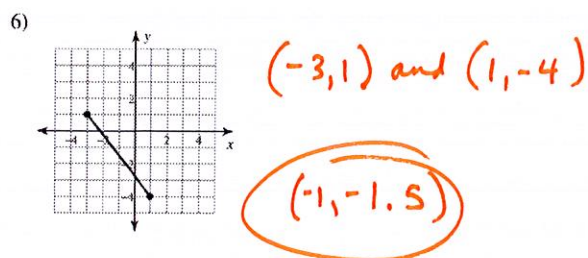
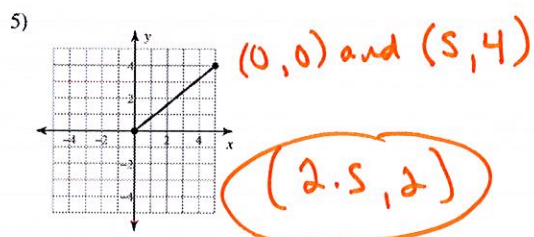
Find the coordinates of Point P on a directed line segment AB in a ratio of 1:2. A(1, 9) B(-5, -3)



$(-1, 5)$

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Directions: Find the midpoint of each segment.



Find the midpoint of the line segment with the given endpoints.

9) $(-4, 4)$, $(5, -1)$
 $(0.5, 1.5)$

10) $(-1, -6)$, $(-6, 5)$
 $(-3.5, -0.5)$

11) $(2, 4)$, $(1, -3)$
 $(1.5, 0.5)$

12) $(-4, 4)$, $(-2, 2)$
 $(-3, 3)$

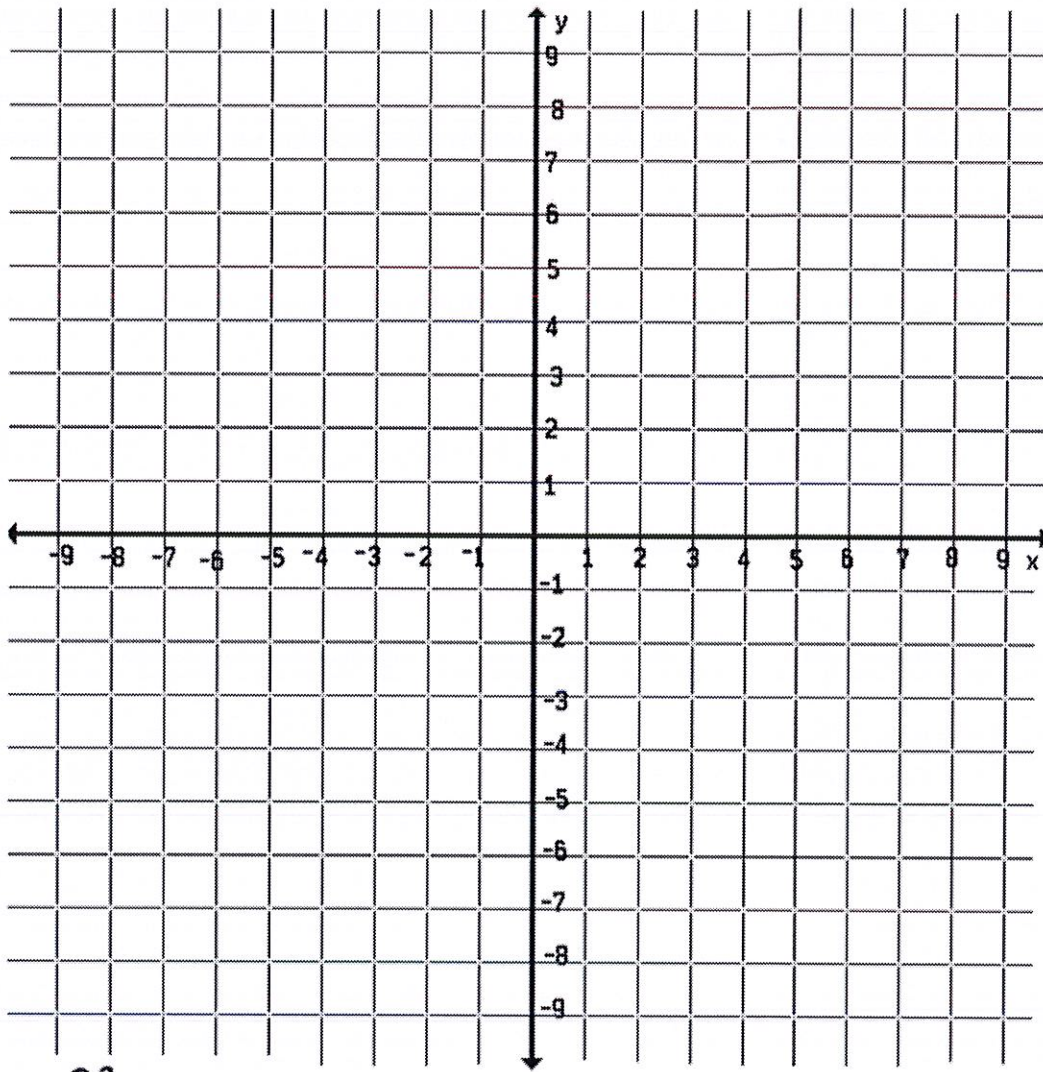
Directions: Find the length of each line segment.

7.) $(3, 7)$ and $(2, 9)$
 $\sqrt{5}$

8.) $(-6, 11)$ and $(4, -1)$
 $\sqrt{244}$

9.) $(0, -8)$ and $(-19, 2)$
 $\sqrt{461}$

10.) $(-4, 2)$ and $(-4, 11)$
 $\sqrt{81} = 9$



Directions: Given two endpoints, partition the line segment in the ratio indicated. Write your answer as a coordinate point.

1. Find the coordinates of point P on directed line segment AB that partition AB in the ratio 1:1.

A (-3, 4) B (7, 6)

(2, 5)

2. Find the coordinates of point P on directed line segment BA that partition BA in the ratio 2:3

A (-9, 3) B (1, 8)

(-3, 6)

3. Find the coordinates of point P on directed line segment AB that partition AB in the ratio 1:3.

A (8, -5) B (4, 7) 1:3

(7, -2)

FINAL REVIEW – Transformations

REFLECTION – 1. a FLIP across an axis of symmetry
2. CONGRUENCE TRANSFORMATION

Reflection in the x-axis:	When you reflect a point across the x -axis, the x -coordinate remains the same, but the y -coordinate is transformed into its opposite. $P(x, y) \rightarrow P'(x, -y)$ or $r_{x\text{-axis}}(x, y) = (x, -y)$
Reflection in the y-axis:	When you reflect a point across the y -axis, the y -coordinate remains the same, but the x -coordinate is transformed into its opposite. $P(x, y) \rightarrow P'(-x, y)$ or $r_{y\text{-axis}}(x, y) = (-x, y)$
Reflection in $y = x$:	When you reflect a point across the line $y = x$, the x -coordinate and the y -coordinate change places. $P(x, y) \rightarrow P'(y, x)$ or $r_{y=x}(x, y) = (y, x)$
Reflection in $y = -x$:	When you reflect a point across the line $y = -x$, the x -coordinate and the y -coordinate change places and are negated (the signs are changed). $P(x, y) \rightarrow P'(-y, -x)$ or $r_{y=-x}(x, y) = (-y, -x)$

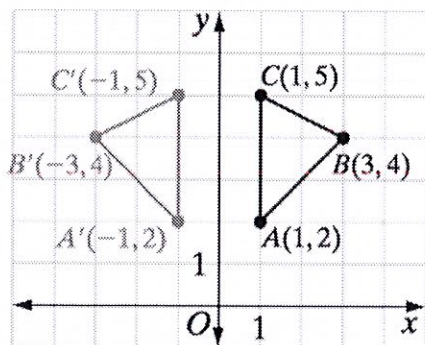
ROTATION: 1. A rotation turns a figure through an angle about a fixed point called the center.
2. CONGRUENCE TRANSFORMATION

Rotation of 90°:	$R_{90^\circ}(x, y) = (-y, x)$
Rotation of 180°:	$R_{180^\circ}(x, y) = (-x, -y)$ (same as point reflection in origin)
Rotation of 270°:	$R_{270^\circ}(x, y) = (y, -x)$

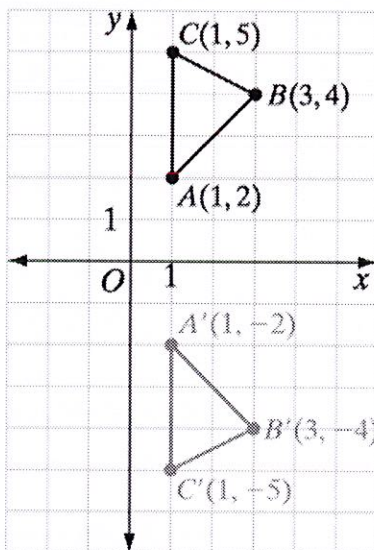
TRANSLATION: 1. A translation "slides" an object a fixed distance in a given direction.
2. CONGRUENCE TRANSFORMATION

Translation of h, k:	$T_{h,k}(x, y) = (x + h, y + k)$
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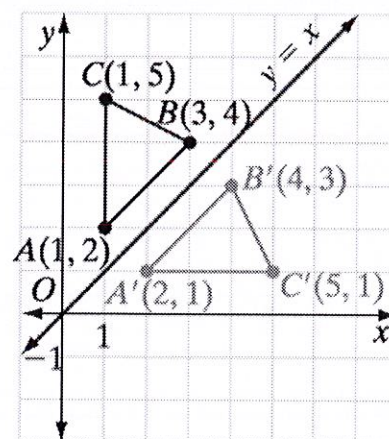
Reflection in the y-axis:



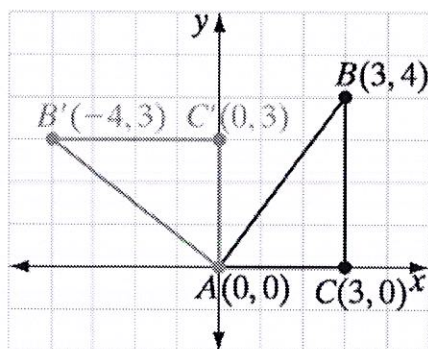
Reflection in the x-axis:



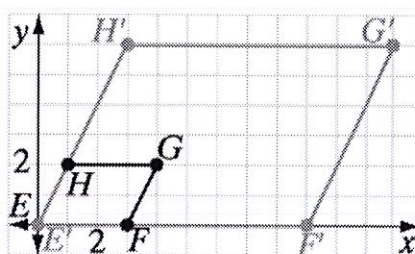
Reflection in $y = x$:



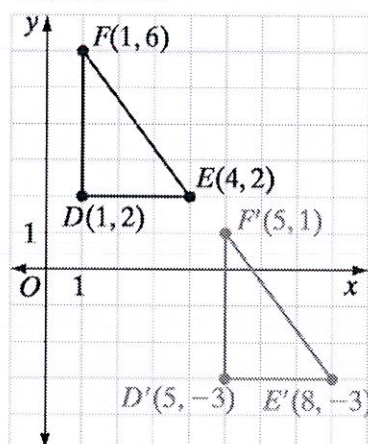
Rotation:
Counter-Clockwise 90-degrees



Dilation (center is $(0, 0)$):

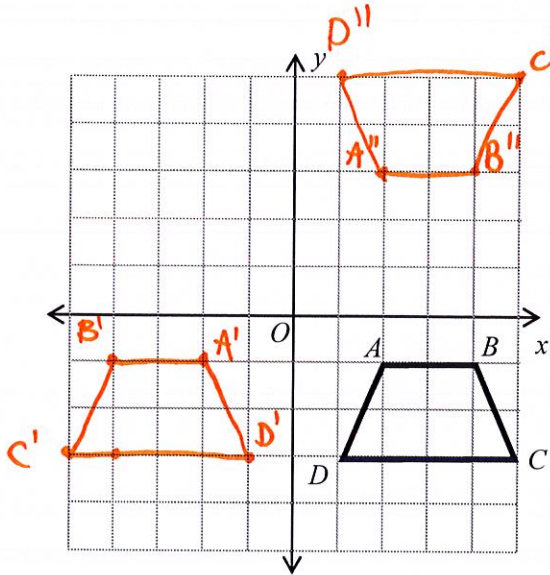


Translation:



- Rotations, reflections, and translations are congruence transformations b/c the pre-image and image remain congruent to each other.
- The pre-image can be mapped onto the image in a congruence transformation.

FINAL REVIEW – Transformations



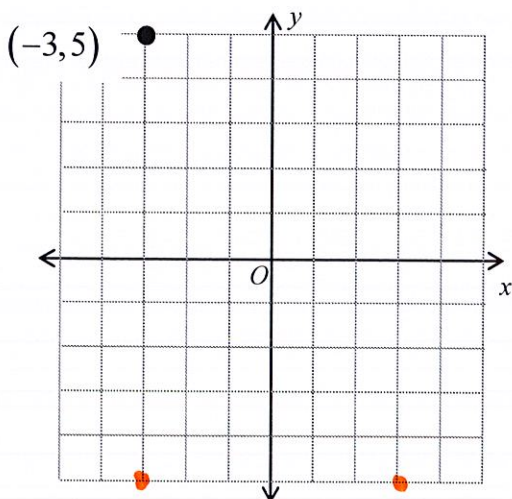
A. Reflect the figure across the y-axis. Name the image $A'B'C'D'$.

B. Does the image have the same side lengths and angle measurements? Justify your answer.

Yes! Reflection is a congruence transformation (the image and pre-image are congruent)

C. If the figure was reflected across the x-axis, then translated two units upward, would the result be the same as the transformation in part a? If not, show where the image would be on the coordinate plane and label it $A''B''C''D''$.

#2



If the pre-image point is $(-3, 5)$, write the coordinates of the image point after it undergoes the following transformations:

a. Reflected across the x-axis $(-3, -5)$

b. Reflected across the y-axis $(3, -5)$

c. Translated seven units to the right and two units downward $[(x+7, y-2)]$ $(10, -7)$

d. Rotated counter-clockwise 90 degrees about the origin.

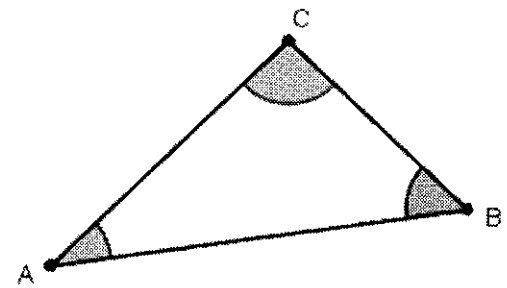
e. Rotated 180 degrees about the origin. $(7, 10)$
 $(-7, -10)$

FINAL REVIEW - Triangles

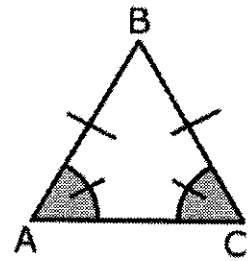
- The sum of the interior angles of ANY triangle is 180° .
- Every triangle has three sides and three angles.

- Classification:

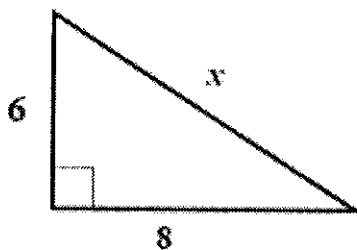
- Equilateral Triangle: all side lengths are equal
 - each interior angle is 60°
- Isosceles Triangle: two side lengths are equal
 - base angles are congruent (see image to the right →)
- Scalene Triangle: no equal side lengths
- Right Triangle: one angle is a right angle, the other two angles are acute (smaller than 90°)



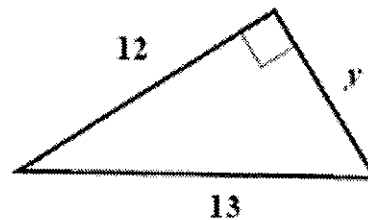
$$m\angle A + m\angle B + m\angle C = 180$$



- PYTHAGOREAN THEOREM! Remember: If you have a right triangle, you can use Pythagorean Theorem to find the missing side. (Examples below)



$$\begin{aligned}6^2 + 8^2 &= x^2 \\36 + 64 &= x^2 \\100 &= x^2 \\\sqrt{100} &= \sqrt{x^2} \\x &= 10\end{aligned}$$

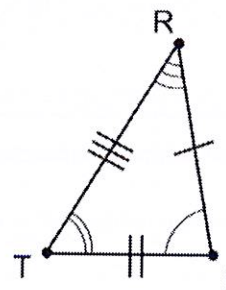
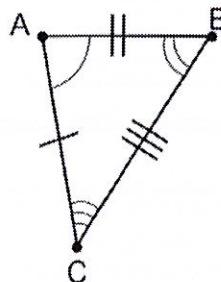
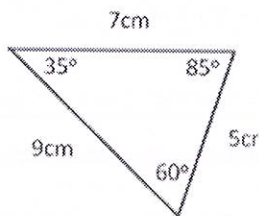
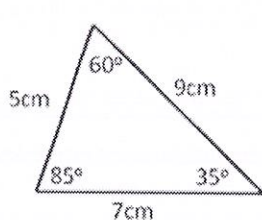


$$\begin{aligned}12^2 + y^2 &= 13^2 \\144 + y^2 &= 169 \\y^2 &= 25 \\\sqrt{y^2} &= \sqrt{25} \\y &= 5\end{aligned}$$

**** NO PROBLEM SET****

FINAL REVIEW - Triangle Congruence

- Two figures are congruent if all **corresponding lengths** are the same, and if all **corresponding angles** have the **same measure**.
- same shape, same size
- Both pairs of triangles below are congruent. You can tell the first pair is congruent because corresponding sides and angles are the same measure. You can tell the second pair is congruent because of the congruence markings.



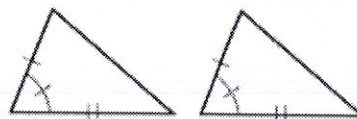
- Proving Triangles Congruent

Side-Side-Side (SSS)



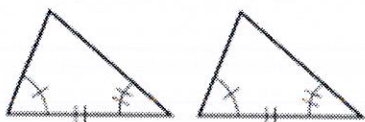
Three pairs of congruent sides

Side-Angle-Side (SAS)



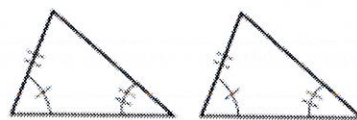
Two pairs of congruent sides and one pair of congruent angles (angles between the pairs of sides)

Angle-Side-Angle (ASA)



Two pairs of congruent angles and one pair of congruent sides (sides between the pairs of angles)

Side-Angle-Angle (SAA)

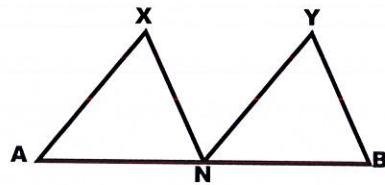


Two pairs of congruent angles and one pair of congruent sides (sides not between the pairs of angles)

CAREFUL! AAA (Angle Angle Angle) and SSA (Side Side Angle) do **NOT** prove two triangles congruent! DO NOT USE THEM IN A PROOF!

- Important Concepts for Proofs

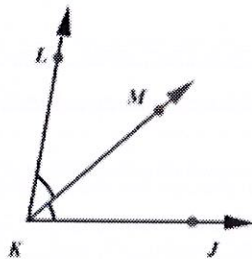
- Midpoint: the middle point of a line segment; It is equidistant from both endpoints; it bisects the segment.



In the diagram to the left:

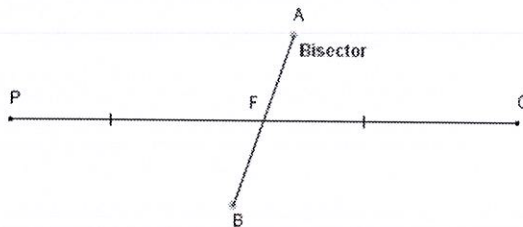
N is the midpoint of \overline{AB} so...
 $\overline{AN} \cong \overline{BN}$

- Bisector: a line that cuts an angle or line segment into two equal parts



In the diagram to the left:

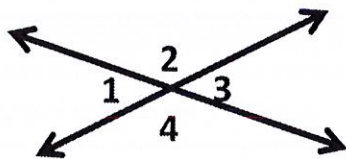
\overrightarrow{KM} bisects $\angle LKJ$ so... $\angle LKM \cong \angle JKM$



In the diagram to the left:

\overline{AB} bisects \overline{PQ} so... $\overline{PF} \cong \overline{QF}$

- Vertical angles



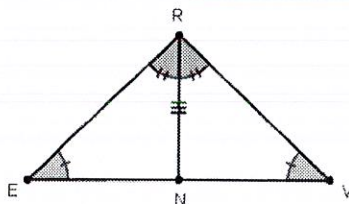
In the diagram to the left:

Angles 1 and 3 are vertical angles so they are CONGRUENT.

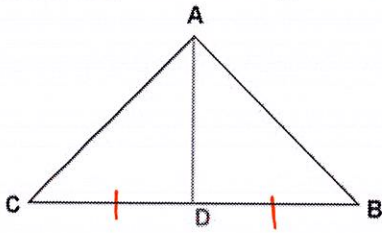
Angles 2 and 4 are vertical angles so they are CONGRUENT.

- Corresponding Parts of Congruent Triangles are Congruent (CPCTC): If two triangles are congruent, then the corresponding parts of those triangles are congruent!

- Reflexive Property – in the diagram below $\overline{RN} \cong \overline{RN}$ by the reflexive property



Directions: Use the diagram to answer the questions below.



\overline{AD} bisects \overline{CB} ; therefore, $\overline{CD} \cong \overline{BD}$. You are asked to prove $\triangle ADC \cong \triangle ADB$.

a. To prove congruence by SSS, what two additional congruence statements are needed?

1. $\overline{AC} \cong \overline{AB}$
2. $(\overline{AD} \cong \overline{AD})$

b. To prove congruence by SAS, what two additional congruence statements are needed?

1. $\angle ADC \cong \angle ADB$
2. $(\overline{AD} \cong \overline{AD})$

c. To prove congruence by ASA, what two additional congruence statements are needed?

1. $\angle C \cong \angle B$
2. $\angle ADC \cong \angle ADB$

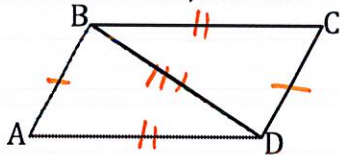
d. To prove congruence by AAS, what two additional congruence statements are needed?

1. $\angle C \cong \angle B$
2. $\angle CAD \cong \angle BAD$

Directions: Fill in the blanks.

1.

Given: $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$



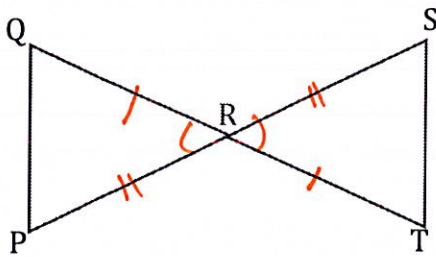
Prove: $\triangle ABD \cong \triangle CBD$

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $\overline{AD} \cong \overline{CB}$	2. Given
3. $\overline{BD} \cong \overline{BD}$	3. Reflexive Prop.
4. $\triangle ABD \cong \triangle CBD$	4. SSS SSS

WLPCS
Geometry

2.

Given: $\overline{QR} \cong \overline{TR}$
 $\overline{PR} \cong \overline{SR}$



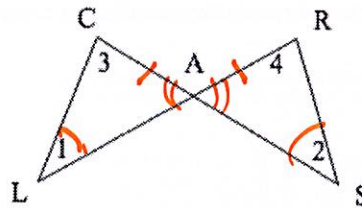
Prove: $\triangle QRP \cong \triangle SRT$

Statements	Reasons
1. $\overline{QR} \cong \overline{TR}$	1. Given
2. $\overline{PR} \cong \overline{SR}$	2. Given
3. $\angle SRT \cong \angle PRQ$	3. vertical angles are congruent
4. $\triangle QRP \cong \triangle SRT$	4. SAS

3.

Given: $\overline{AC} \cong \overline{AR}$ and $\angle 1 \cong \angle 2$

Prove: $\angle 3 \cong \angle 4$



Proof:

1. $\overline{AC} \cong \overline{AR}$

2. $\angle 1 \cong \angle 2$

3. $\angle CAL \cong \angle RAS$

4. $\triangle LCA \cong \triangle RSA$

5. $\angle 3 \cong \angle 4$

1. Given

2. Given

3. vertical angles are congruent

4. AAS

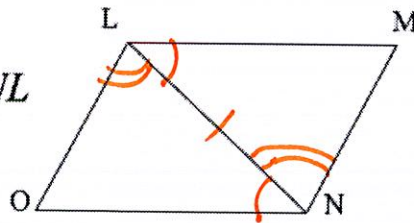
5. CPCTC

(corresponding parts of congruent triangles are \cong)

4.

Given: $\angle NLM \cong \angle LNO$ and $\angle OLN \cong \angle MNL$

Prove: $\angle M \cong \angle O$



Proof:

1. $\angle NLM \cong \angle LNO$

2. $\angle OLN \cong \angle MNL$

3. $\overline{LN} \cong \overline{LN}$

4. $\triangle LMN \cong \triangle NLM$

5. $\angle M \cong \angle O$

1. Given

2. Given

3. Reflexive Property of \cong

4. ASA

5. CPCTC

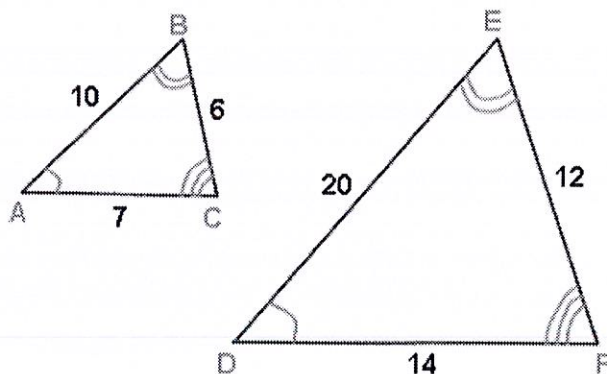
FINAL REVIEW: Similarity and Dilations

Definition 1: Two figures are similar if you can map one onto the other through a series of translations, reflections, rotations AND DILATIONS (so you can scale up and down).

Definition 2: Two figures are similar if their **corresponding angles are congruent** and the **ratio of their corresponding sides are equal (proportional side lengths)**.

Definition 3: **Same shape**, not necessarily same size!

* This is true for ALL polygons, not just triangles. *



$\triangle ABC \sim \triangle DEF$ because...

1. ALL corresponding angles are congruent
2. ALL corresponding sides are in the same ratio (2:1).

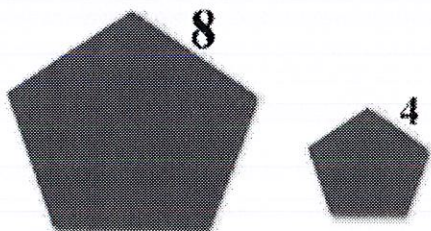
Finding a scale factor...

Since **similar** figures are really just **dilations**, we can find **scale factors** in the same way.

Decide if it is an enlargement or reduction and put **corresponding sides in a RATIO**. Make sure that ratio is **LARGER** than 1 if it is an **enlargement** and **SMALLER** than 1 if it is a **reduction**.

For example: From Pentagon A to B \rightarrow reduction and a scale factor of $\frac{4}{8}$ or $\frac{1}{2}$ or 0.5

From Pentagon B to A \rightarrow enlargement and a scale factor of $\frac{8}{4}$ or 2.



To find an unknown side...

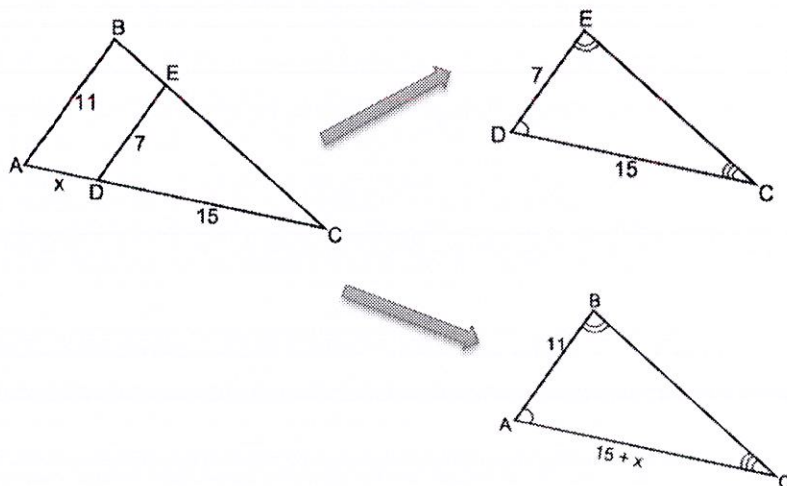
Apply the scale factor to set up a proportion!

$$\frac{1}{2} = \frac{3}{x}$$

WLPCS
Geometry

Sometimes similar figures may overlap...

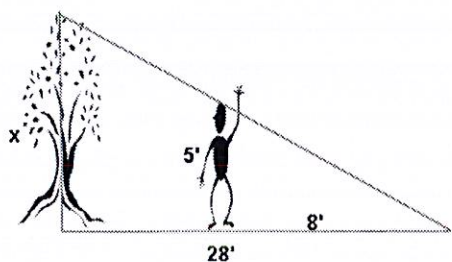
In the diagram below $\triangle ABC \sim \triangle DEC$. It can be separated into TWO triangles (if that is helpful). See below.



To solve for x you still set up a proportion starting with the **scale factor**:

$$\frac{7}{11} = \frac{15}{15 + x}$$

Shadows can create similar triangles...

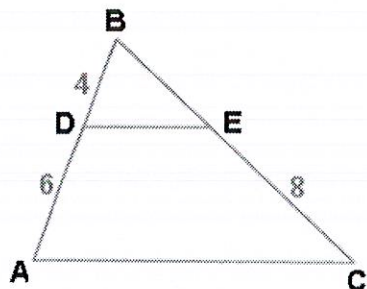


One way to set up a proportion:

$$\frac{\text{height of man}}{\text{height of tree}} = \frac{\text{length of man's shadow}}{\text{length of tree's shadow}}$$

$$\frac{5}{x} = \frac{8}{28}$$

Proving triangles similar...

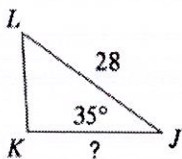


If you are given that $\overline{DE} \parallel \overline{AC}$, you can prove that the triangles are similar in the following steps:

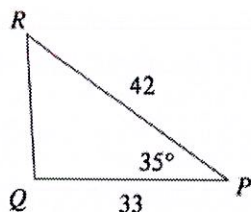
1. $\angle B \cong \angle B$ (reflexive property)
2. $\angle BDE \cong \angle BAC$ (corresponding angles... remember \overline{DE} and \overline{AC} are parallel!!!)
3. This is enough! If you can prove two sets of corresponding angles are congruent, you're done!

WLPCS
Geometry

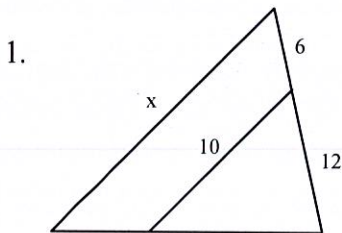
Directions: If $\triangle LKJ \sim \triangle RQP$, what is the length of \overline{KJ} ?



$$\frac{28}{42} = \frac{x}{33}$$



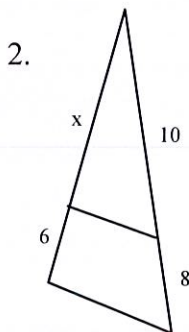
Directions: Find the length of x .



$$\frac{12}{18} = \frac{10}{x}$$

$$12x = 180$$

$$x = 15$$



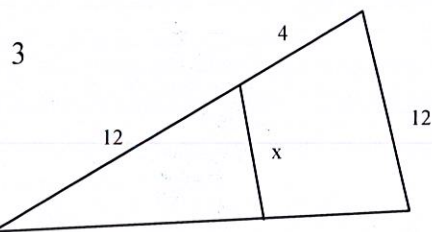
$$\frac{x}{x+6} = \frac{10}{18}$$

$$18x = 10x + 60$$

$$-10x - 10x$$

$$8x = 60$$

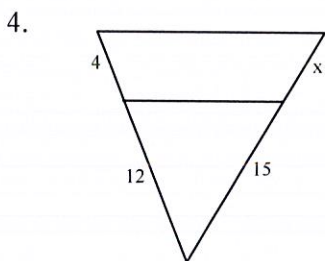
$$x = 7.5$$



$$\frac{12}{16} = \frac{x}{12}$$

$$144 = 16x$$

$$9 = x$$



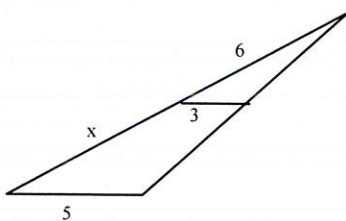
$$\frac{12}{16} = \frac{15}{15+x}$$

$$240 = 180 + 12x$$

$$60 = 12x$$

$$5 = x$$

5. $x = 7.5$

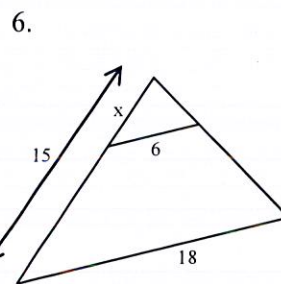


$$\frac{6}{6+x} = \frac{3}{5}$$

$$30 = 18 + 3x$$

$$12 = 3x$$

$$4 = x$$



$$\frac{x}{15} = \frac{6}{18}$$

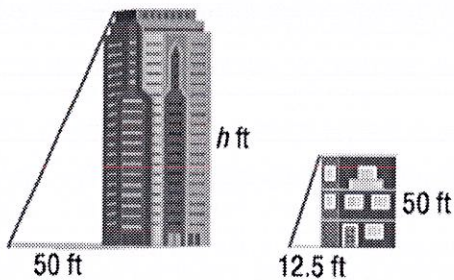
$$18x = 90$$

$$x = 5$$

WLPCS
Geometry

Directions: Find the height of the building given the shadow lengths and the height of the smaller building.

** shadow problems → similar Δs **



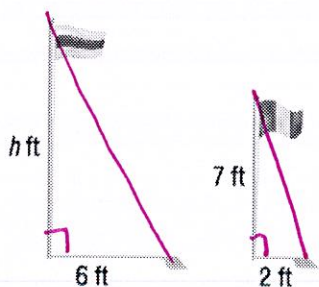
$$\frac{12.5}{50} = \frac{50}{h}$$

$$2500 = 12.5h$$

$$200 = h$$

$$\underline{200 \text{ ft.}}$$

Directions: Find the height of the taller flagpole.



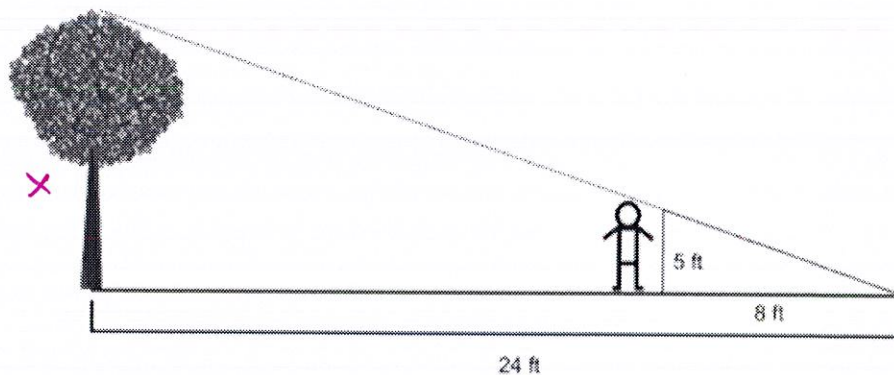
$$\frac{6}{2} = \frac{h}{7}$$

$$\frac{42}{2} = \frac{2h}{2}$$

$$\underline{h = 21}$$

$$\underline{21 \text{ ft.}}$$

Directions: Find the height of the tree.



$$\frac{5}{8} = \frac{x}{24}$$

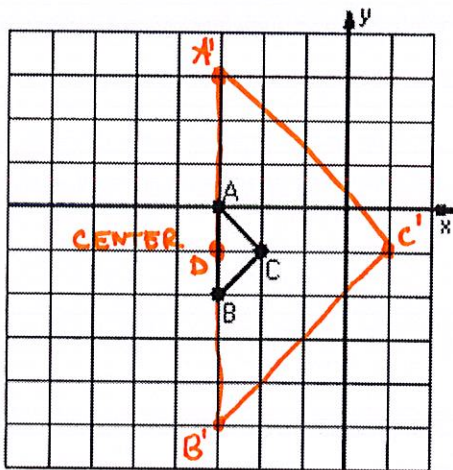
$$120 = 8x$$

$$\underline{15 = x}$$

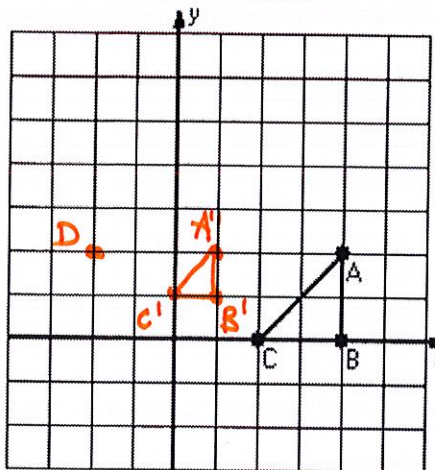
$$\underline{15 \text{ ft.}}$$

Directions: Perform the dilation according to the scale factor and center of dilation.

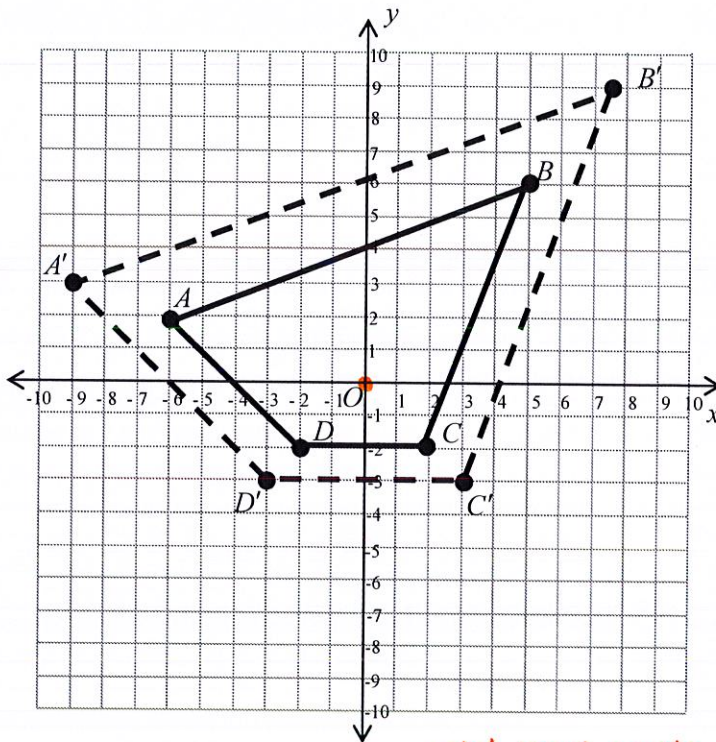
3) Dilation scale = 4, center D(-3,-1)



4) Dilation scale = $\frac{1}{2}$, center D(-2,2)



On the graph below, quadrilateral ABCD has been dilated, with the center of dilation at the origin, to create quadrilateral A'B'C'D'.



a. What is the scale factor of the dilation?

*D'C' is 6 units, DC is 4 units
scale factor: $\frac{6}{4}$ or $\frac{3}{2}$ or 1.5*

b. What is the ratio of the length of $\overline{A'B'}$ to the length of \overline{AB} ?

A'B' : AB 3:2

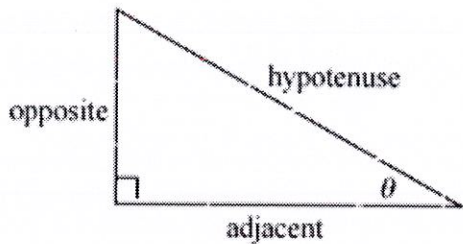
c. How are the measures of $\angle A$ and $\angle A'$ related?

They are congruent!

FINAL REVIEW – Trigonometry

We will focus on **right triangle trigonometry**... so we can only work with **right triangles**.

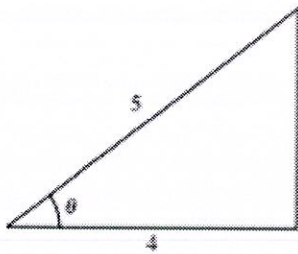
There are **three** trigonometric functions we have worked with. Each is a **ratio**.



$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$



$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$$

How to use inverse trig. functions:

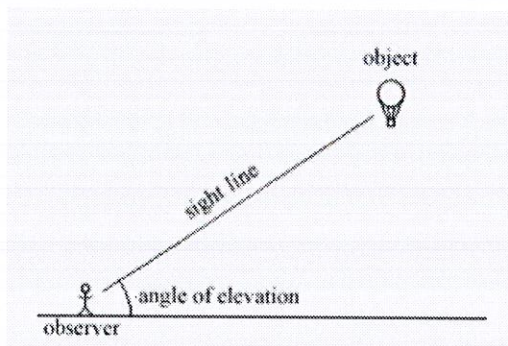
$$\sin \theta = \frac{3}{5}$$

$$\theta = 37^\circ$$

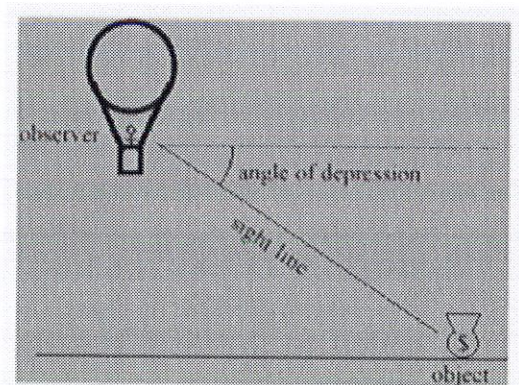
$$\theta = \sin^{-1}\left(\frac{3}{5}\right)$$

** inverse trig. functions
help you find unknown
angles **

Angle of Elevation – The angle formed by the horizontal and the line of sight above the horizontal (to see an object that is higher than the observer).



Angle of Depression – The angle formed by the horizontal and the line of sight below the horizontal (to see an object that is lower than the observer).



Directions: Complete on a separate sheet of paper. Be sure to sketch a diagram!

Example 1 - Find to the nearest degree the measure of the angle of elevation of the sun when a vertical pole 6.5 meters high casts a shadow 8.3 meters long.

Example 2 - From the top of a lighthouse 165 feet above sea level, the measure of the angle of depression of a boat at sea is 35° . Find to the nearest foot the distance from the boat to the foot of the lighthouse.

Example 3 - At a point on the ground 39 meters from the foot of a tree, the measure of the angle of elevation of the top of the tree is 42° . Find the height of the tree to the nearest meter.

Example 4: From the top of a school 61 feet high, the measure of the angle of depression to the road in front of the school is 38° . Find to the nearest foot the distance from the road to the school.

5)

A ladder is leaning against a wall. The foot of the ladder is 6.25 feet from the wall. The ladder makes an angle of 74.5° with the level ground. How high on the wall does the ladder reach? Round the answer to the *nearest tenth of a foot*.

6)

Find to the *nearest degree* the measure of the angle of elevation of the sun when a woman 150 centimeters tall casts a shadow 43 centimeters long.

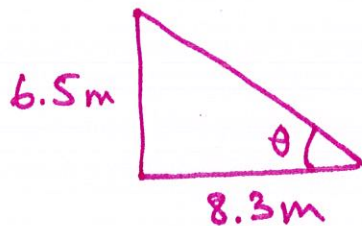
7)

3. A person in a balloon which is 2,000 feet above the airport finds that the angle of depression to a ship out at sea is 21° . Find the horizontal distance between the balloon and the ship. (or the distance from the airport to the ship)

8)

Find the angle of elevation of the sun when a 24 foot tree casts a shadow of 36

Ex 1:

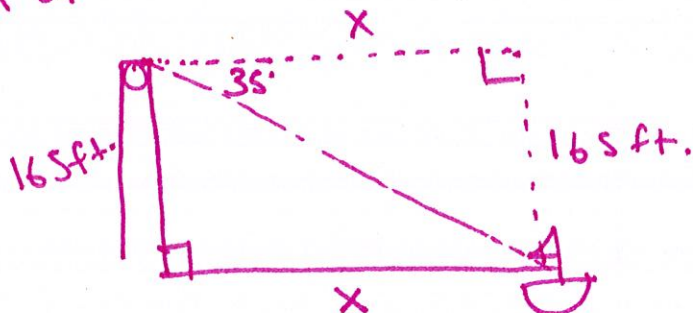


$$\tan \theta = \frac{6.5}{8.3}$$

$$\theta = \tan^{-1} \left(\frac{6.5}{8.3} \right)$$

$$\theta = 38^\circ$$

Ex 2:



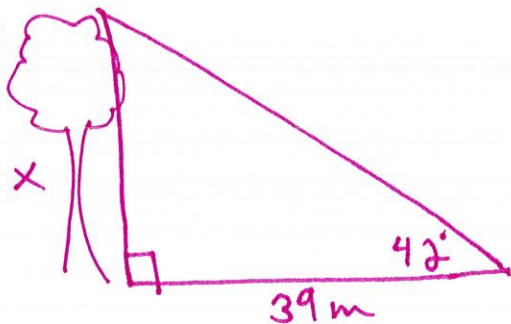
$$\tan 35 = \frac{165}{x}$$

Multiply each side by x to get:

$$\frac{x(\tan 35) = 165}{\tan 35 \quad \tan 35}$$

$$x = 236 \text{ ft.}$$

Ex 3:



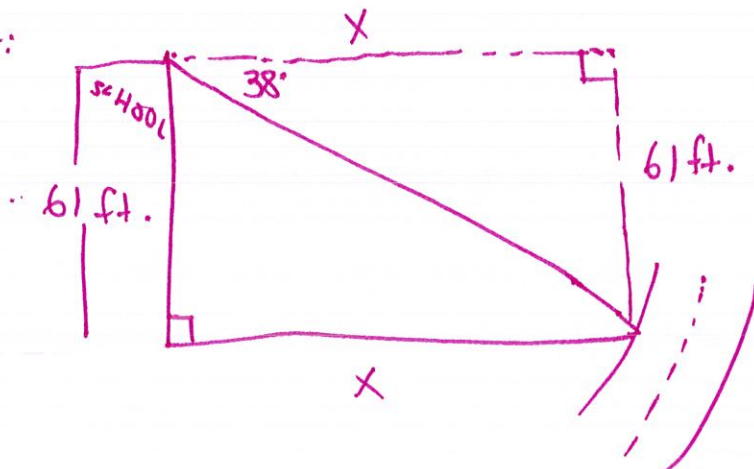
$$\tan 42^\circ = \frac{x}{39}$$

Multiply by 39 on each side:

$$x = (\tan 42^\circ) 39$$

$$x = 35 \text{ m}$$

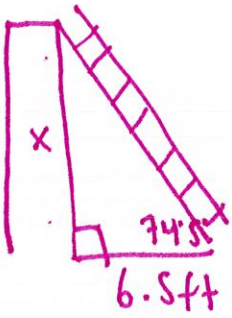
Ex 4:



$$\tan 38^\circ = \frac{61}{x}$$

$$x = 78 \text{ ft.}$$

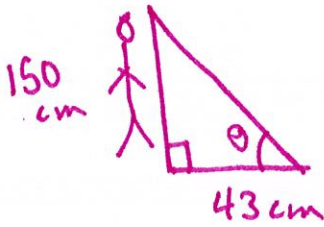
Ex 5:



$$\tan 74.5^\circ = \frac{x}{6.25}$$

$$x = 22.5 \text{ ft}$$

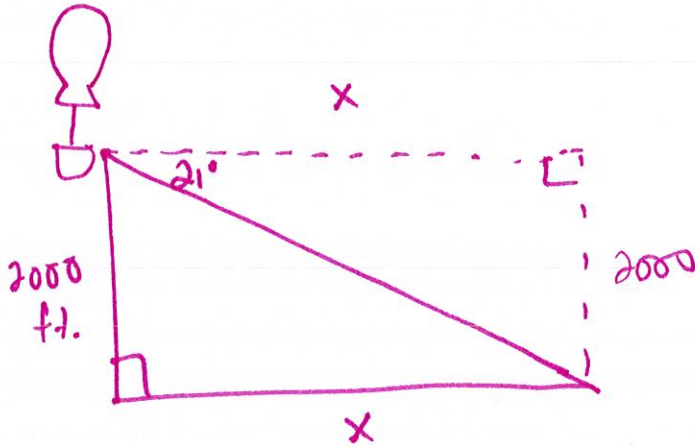
Ex 6:



$$\tan \theta = \frac{150}{43}$$

$$\theta = 74^\circ$$

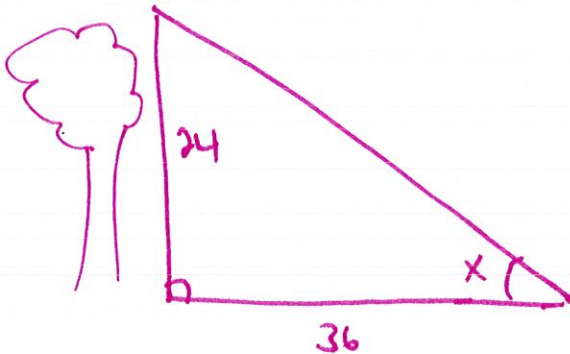
Ex 7:



$$\tan 21^\circ = \frac{2000}{x}$$

$$x = 5,210 \text{ ft}$$

Ex 8:

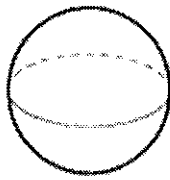


$$\tan x = \frac{24}{36}$$

$$x = 34^\circ$$

FINAL REVIEW – Area and Volume

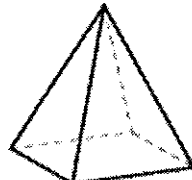
- Geometric solids:



sphere



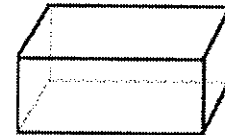
cone



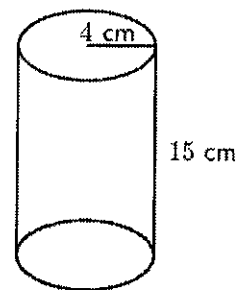
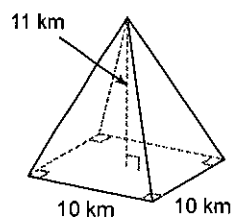
pyramid



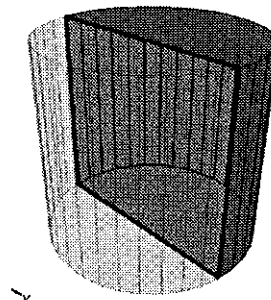
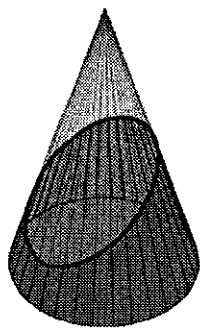
cylinder



rectangular prism

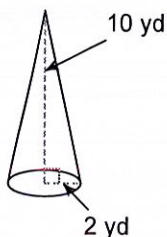


- Cross-section: the 2-D figure formed by the intersection of a plane and a 3-D solid



Directions: Find the volume of each figure. USE PROPER UNITS.

* 13)

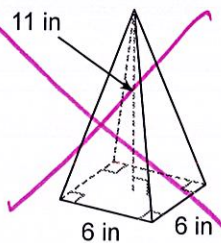


$$\frac{1}{3} \pi (2)^2 (10)$$

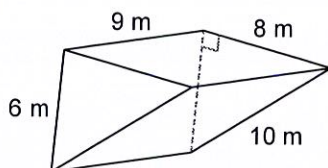
$$\frac{1}{3} \pi 40$$

$$41.9 \text{ yd}^3$$

14)



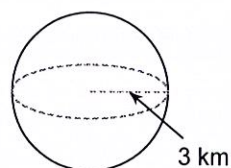
15)



$$\frac{1}{2} \cdot 6 \cdot 8 \cdot 9$$

$$216 \text{ m}^3$$

* 16)



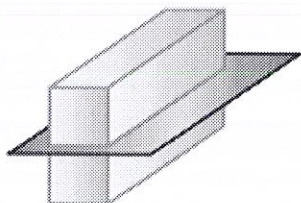
$$\frac{4}{3} \pi (3)^3$$

$$113.04 \text{ km}^3$$

Directions: Name the figure AND identify the shape of the cross-section.

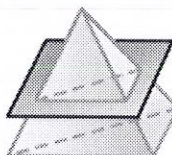
13.

• prism
• rectangle



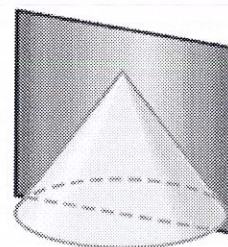
14.

• pyramid
• triangle



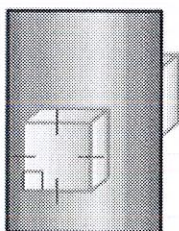
15.

• cone
• triangle



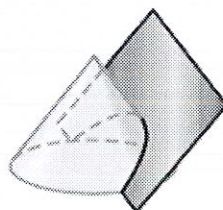
16.

• prism
• square



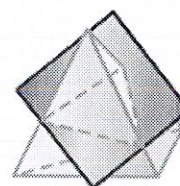
17.

• cone
• parabola



18.

• pyramid
• triangle



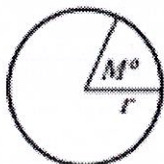
FINAL REVIEW – Circles

Circle Formulas:

Shape	Area	Circumference
Circle	$A = \pi r^2$	$C = \pi d = 2\pi r$

Arcs and sectors:

Arc and Sector

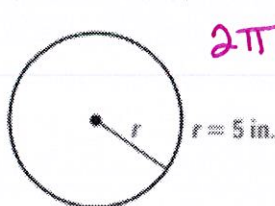


$$\text{Arc Length} = \left(\frac{M}{360}\right) \cdot 2\pi r$$

$$\text{Sector Area} = \left(\frac{M}{360}\right) \cdot \pi r^2$$

USING CIRCUMFERENCE In Exercises 15 and 16, find the indicated measure.

15. Circumference

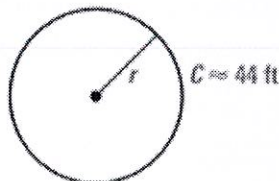


$$2\pi(5)$$

$$10\pi \text{ in}$$

$$31.4 \text{ in}$$

16. Radius



$$\frac{44}{2\pi} = \frac{2\pi r}{2\pi}$$

$$7\text{ft} = r$$

17. Find the circumference of a circle with diameter 8 meters. $8\pi \text{ m}$ or 25.12 m

18. Find the circumference of a circle with radius 15 inches. (Leave your answer in terms of π .) $30\pi \text{ in.}$

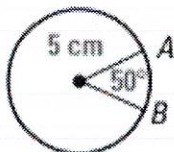
19. Find the radius of a circle with circumference 32 yards.

$$\frac{32}{2\pi} = \frac{2\pi r}{2\pi}$$

$$r = 5.1 \text{ yds}$$

Find the length of each arc.

a.



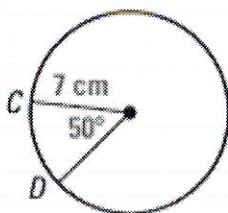
$$\frac{50}{360} \cdot 2\pi(5\text{cm})$$

$$10.9 \text{ cm}^2$$

$$\frac{50}{360} \cdot 2\pi(5\text{cm})$$

$$4.36 \text{ cm}$$

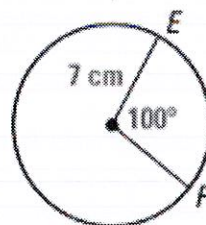
b.



$$\frac{50}{360} \cdot 2\pi(7\text{cm})$$

$$6.1 \text{ cm}$$

c.



$$\frac{100}{360} \cdot 2\pi(7\text{cm})$$

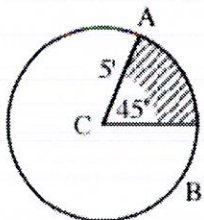
$$12.2 \text{ cm}$$

WLPCS
Geometry

Area/Circumference Matching:

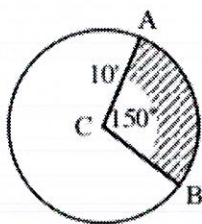
- | | | |
|----------|--|-----------|
| <u>b</u> | 1. Patricia buys a round dinner table. The radius of dinner table is 6 meters. What is the area of table? | a. 56.52 |
| <u>f</u> | 2. Charles has a circular carpet in his drawing room. He wants to put a circular table in middle of the carpet. The diameter of the carpet is 12 meters and the diameter of the table is 4 meters. Calculate how much area of carpet is left after putting the table in place? | b. 113.04 |
| <u>c</u> | 3. Steven purchases a bowl. The diameter of the bowl is 14 cm. What is the circumference of the bowl? | c. 43.96 |
| <u>g</u> | 4. Daniel wants to buy cookies for her friend. The radius of a cookie is 5 inches. What is the cookie's circumference? | d. 50.24 |
| <u>d</u> | 5. Donna makes a round pizza. She wants to put a cheese layer on the pizza. If the cake is 8 cm in diameter, how many square cm of cheese layer does she need to put on the pizza? | e. 19.625 |
| <u>a</u> | 6. Cynthia wants to buy a round photo frame for her brother. The radius of the photo frame is 9 cm. What is the photo frame's circumference? | f. 100.48 |
| <u>e</u> | 7. Brian made a tasty burger. The diameter of the burger was 5 cm. What was the area of burger? | g. 31.4 |

Find the area of the 45° sector.



$$\frac{45}{360} \cdot \pi (5)^2 = 9.8 \text{ ft}^2$$

Find the area of the 150° sector.



$$\frac{150}{360} \cdot \pi (10)^2 = 130.8 \text{ ft}^2$$

Circle Equations:

Circles Review Sheet

- 1) Write an equation for the circle with center $(-4, 1)$ and radius 5.

$$(x - (-4))^2 + (y - 1)^2 = 5^2$$

$$(x + 4)^2 + (y - 1)^2 = 25$$

- 2) Write an equation for the circle with center $(0, -3)$ and radius 9.

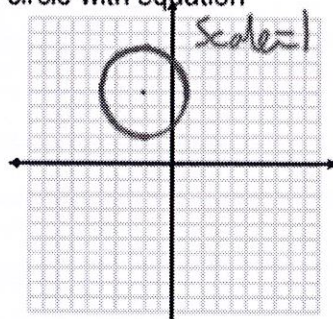
$$(x - 0)^2 + (y - (-3))^2 = 9^2$$

$$x^2 + (y + 3)^2 = 81$$

- 3) Find the center and radius of the circle with equation

$$(x + 2)^2 + (y - 5)^2 = 9 \text{ and graph it.}$$

Center: $(-2, 5)$
Radius: $= \sqrt{9} = 3$



USING STANDARD EQUATIONS Give the center and radius of the circle.

7. $(x - 4)^2 + (y - 3)^2 = 16$

8. $(x - 5)^2 + (y - 1)^2 = 25$

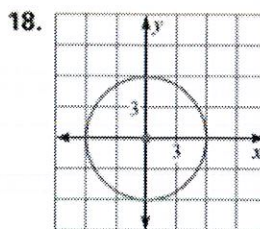
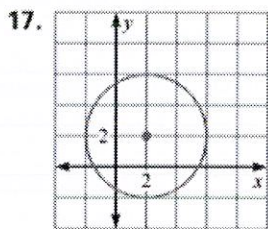
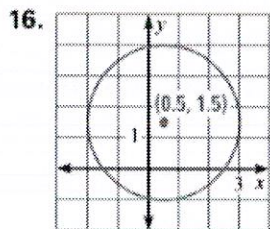
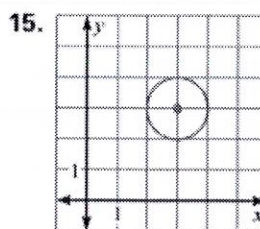
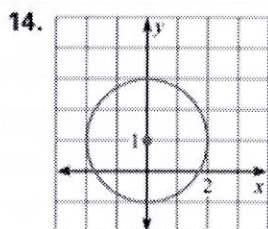
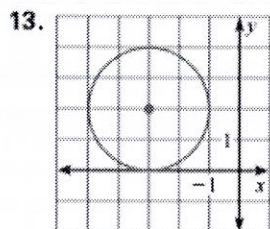
9. $x^2 + y^2 = 4$

10. $(x + 2)^2 + (y - 3)^2 = 36$

11. $(x + 5)^2 + (y + 3)^2 = 1$

12. $(x - \frac{1}{2})^2 + (y + \frac{3}{4})^2 = \frac{1}{4}$

USING GRAPHS Give the coordinates of the center, the radius, and the equation of the circle.



WRITING EQUATIONS Write the standard equation of the circle with the given center and radius.

19. center $(0, 0)$, radius 1

20. center $(4, 0)$, radius 4

21. center $(3, -2)$, radius 2

22. center $(-1, -3)$, radius 6

ANSWERS:

7. $(4, 3)$ radius: 4

8. $(5, 1)$ radius: 5

9. $(0, 0)$ radius: 2

10. $(-2, 3)$ radius: 6

11. $(-5, -3)$ radius: 1

12. $(\frac{1}{2}, -\frac{3}{4})$ radius: $\frac{1}{2}$

13. $(x + 3)^2 + (y - 2)^2 = 4$

14. $(x)^2 + (y - 1)^2 = 4$

15. $(x - 3)^2 + (y - 3)^2 = 1$

16. $(x - 0.5)^2 + (y - 1.5)^2 = 6.25$

17. $(x - 2)^2 + (y - 2)^2 = 4$

18. $x^2 + y^2 = 4$

19. $x^2 + y^2 = 1$

20. $(x - 4)^2 + y^2 = 16$

21. $(x - 3)^2 + (y + 2)^2 = 4$

22. $(x + 1)^2 + (y + 3)^2 = 36$